# Pairwise-optimal Discrete Coverage Control for Gossiping Robots

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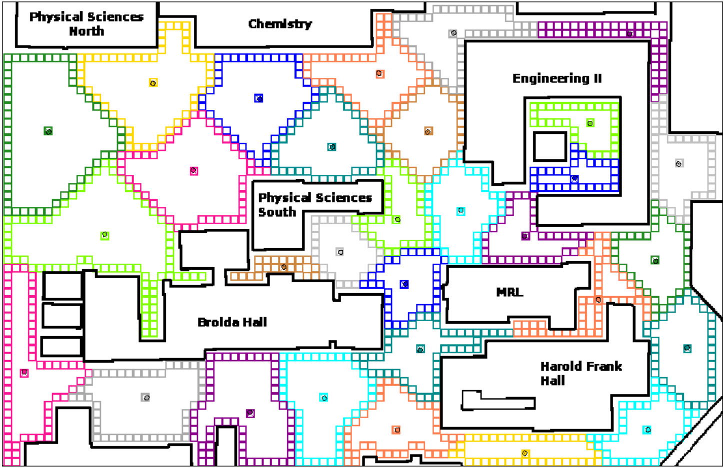
Joint work with Ruggero Carli and Francesco Bullo

# The Coverage Control Problem




***The Team:*** Robots with limited communciation capabilities

***The Mission:*** Spatially distributed load balancing

# Overview

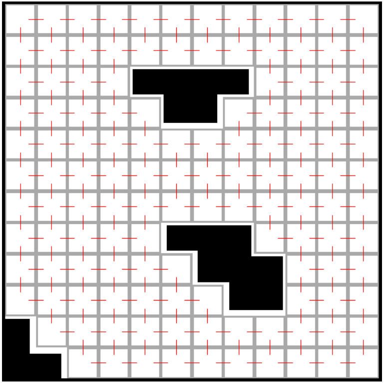
*Challenges in Coverage Control:*

- Reduce communication requirements
- Ensure solutions scales well for large teams
- Avoid local minima solutions

*This work:*

- Gossip communication: asynchronous, pairwise
- Computation scales with local partition size
- Better solution set than existing methods

# Discretized Coverage



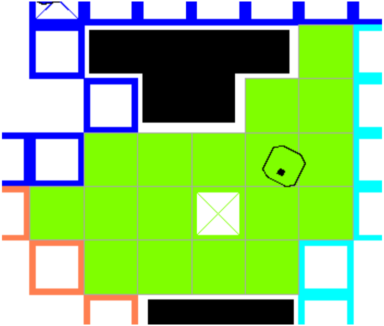
Domain is a weighted graph $G = (Q, E, w)$

***Required properties***

- $G$ must be connected
- All edge-weights $w$ must be positive

Partition vertices $Q$ into regions for each robot: $\{P_1, \ldots, P_n\}$ such that each $P_i$ induces connected subgraph

# Discretized Coverage Cost Function



***Per agent cost***

Cost to cover $P_i$ from vertex $h \in P_i$:

$$H_i(h, P_i) = \sum_{k \in P_i} \mathrm{dist}_{P_i}(h, k)\, \phi(k)$$

***Centroid:*** $c_i \in P_i$ is vertex which minimizes $H_i(h, P_i)$

## Total coverage cost

$$\mathcal{H}_{\text{multi-center}}(c, P) = \sum_{i=1}^{N} H_i(c_i, P_i)$$

# Prior Work: Lloyd Algorithm

**Theorem (Lloyd '57 "least-square quantization")**

1. *at fixed partition, optimal positions are centroids*
2. *at fixed positions, optimal partition is Voronoi*

***Lloyd's Algorithm***

For each update round:

1: move agents towards centroids of their regions
2: take Voronoi partition generated by agent positions

Result: convergence to set of centroidal Voronoi partitions
(i.e., positions are centroids of Voronoi partition they generate)

# Prior Work: Gossip Lloyd Optimization

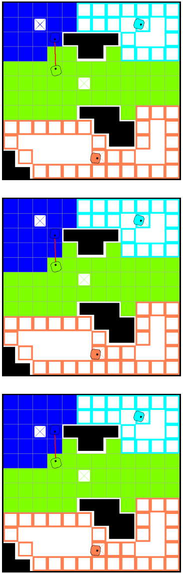
### *Gossip Lloyd optimization*

Whenever two robots communicate:

1: Compute union of regions
2: Perform Voronoi partition of union using prior centroids
3: Update centroid of each agent for new region

Result: convergence to a centroidal Voronoi partition

(Durham, Frasca, Carli, Bullo '09)



# Observation

For pairwise updates: no need to use Lloyd separation!

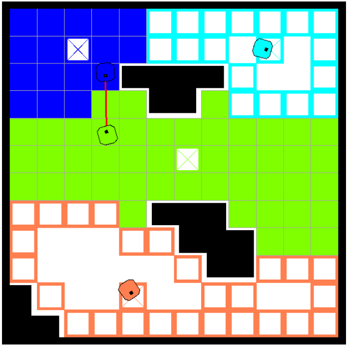
$\Rightarrow$ Finding optimal two partition of graph is not computationally hard

# New Algorithm

### *Pairwise-optimal Coverage*

Whenever two robots communicate:

```
Compute union of regions
for sample vertex pair (a, b) do
    Perform Voronoi parition of union using (a, b)
    if Cost is lower then
        Update centroids to (a, b)
        Update regions
    end if
end for
```

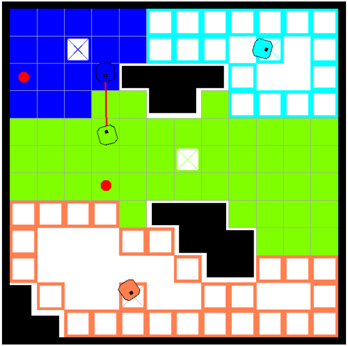


# New Algorithm

### *Pairwise-optimal Coverage*

Whenever two robots communicate:

1: Compute union of regions
2: **for** sample vertex pair $(a, b)$ **do**
3: Perform Voronoi parition of union using $(a, b)$
4: **if** Cost is lower **then**
5: Update centroids to $(a, b)$
6: Update regions
7: **end if**
8: **end for**



# New Algorithm

### *Pairwise-optimal Coverage*

Whenever two robots communicate:

1: Compute union of regions
2: **for** sample vertex pair $(a, b)$ **do**
3: Perform Voronoi parition of union using $(a, b)$
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7: **end if**
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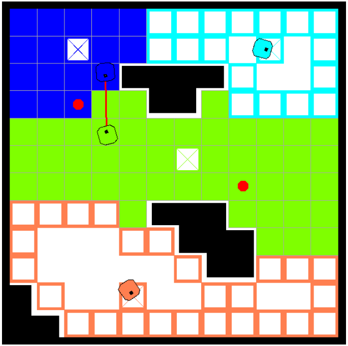
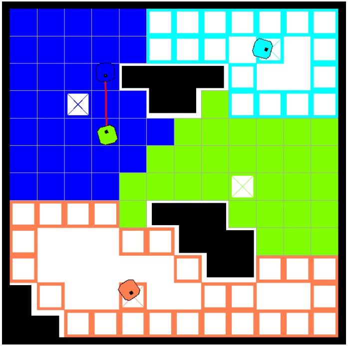


# New Algorithm

### *Pairwise-optimal Coverage*

Whenever two robots communicate:

Compute union of regions<br>
**for** sample vertex pair $(a, b)$ **do**<br>
    Perform Voronoi parition of union using $(a, b)$<br>
    **if** Cost is lower **then**<br>
        Update centroids to $(a, b)$<br>
        Update regions<br>
    **end if**<br>
**end for**



# Scalability Properties

### *Pairwise-optimal Coverage*

Whenever two robots communicate:

```
Compute union of regions
for sample vertex pair (a, b) do
    Perform Voronoi parition of union using (a, b)
    if Cost is lower then
        Update centroids to (a, b)
        Update regions
    end if
end for
```

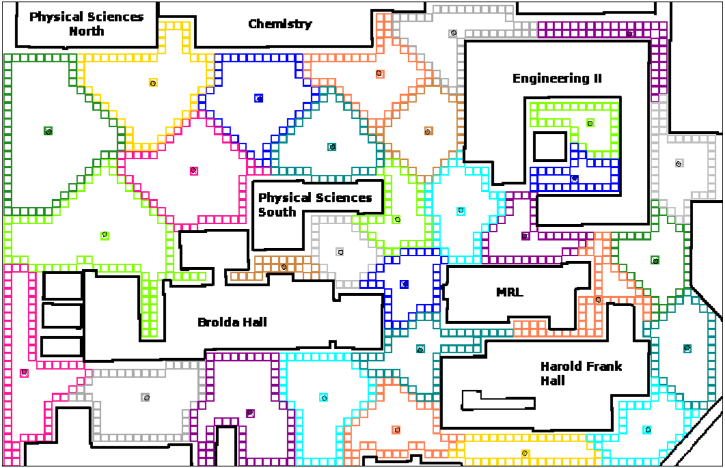
Let $k = |P_i \cup P_j|$:

$\rightarrow$ From 1 to $\mathcal{O}(k^2)$ samples

$\rightarrow$ Finding vertex distances is either $\mathcal{O}(k)$ or $\mathcal{O}(k \log(k))$

$\Rightarrow$ Algorithm is incremental, can be truncated if need be

# Simulation Movie



# Main Convergence Result

**Theorem (Convergence under persistent gossip)**

*If there exists a positive probability for any pairwise exchange in a finite time window, then the evolutions of $c$ and $P$ converge almost surely to a pairwise-optimal partition in finite time.*

***Proof sketch***

1) Regions remain connected
2) Total cost decreases with every pairwise update
3) Pairwise-optimal partitions are equilibrium set

# Pairwise Optimal Partitions

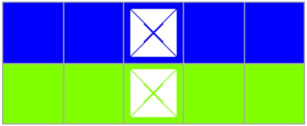
***Definition:*** A partition is a pairwise-optimal partition if, for every pair of neighboring robots $(i, j)$:

$$H_i(c_i; P_i) + H_j(c_j; P_j) = \min_{a,b \in p_i \cup p_j} \left\{ \sum_{k \in P_i \cup P_j} \min \left\{ d_{P_i \cup P_j}(a, k), d_{P_i \cup P_j}(b, k) \right\} \right\}$$

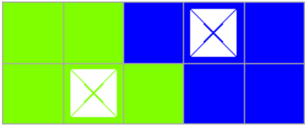
i.e., $(i, j)$ has reached lowest possible coverage cost of $P_i \cup P_j$

$\Rightarrow$ Every pairwise-optimal partition is also centroidal Voronoi

# Subset of Centroidal Voronoi



Cost: 12 hops

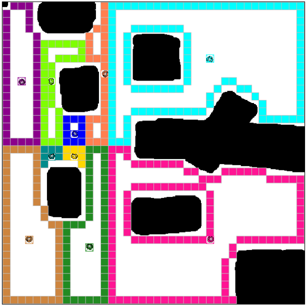


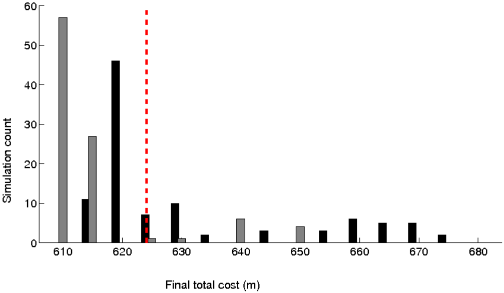
Cost: 10 hops

- Both are centroidal Voronoi
- Only lower cost is pairwise-optimal

$\Rightarrow$ Avoid all pairwise local minima

# Statistical Results I



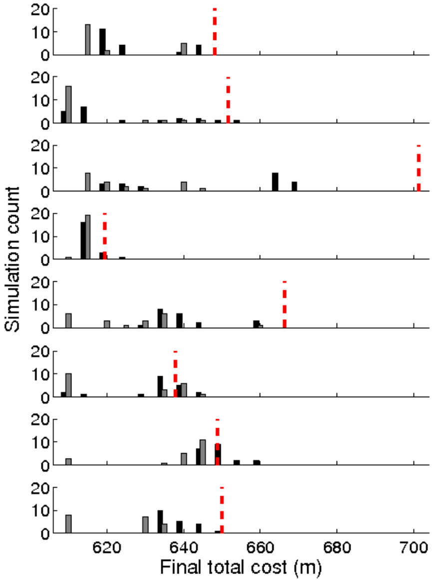



Initial cost: 1,032 *m*

100 sequences of pairwise exchanges

Gray - Pairwise-optimal Coverage
Black - Gossip Lloyd Coverage
Red - Centralized Lloyd Coverage

# Statistical Results II




- 8 random initial conditions
- 20 sequences of pairwise exchanges

Gray - Pairwise-optimal Coverage
Black - Gossip Lloyd Coverage
Red - Centralized Lloyd Coverage

# Summary

### *Chief contributions*

- Pairwise-optimal gossip coverage control algorithm
- Flexible computational requirement
- Improved performance over Lloyd-type methods

### *Future directions*

- Dynamic teams and environments
- Pairwise optimization approach seems useful for wide class of problems